The Origin of Magnetic Storms.

By ARTHUR SCHUSTER, F.R.S.

(Received January 9,—Read January 26, 1911.)

1. Lord Kelvin,* in discussing the origin of magnetic storms, came to the conclusion that they could not be due to a direct solar action on account of the enormous energy which would have to be supplied by the sun. This verdict was generally accepted until recently, when the theory of a direct solar action has been revived in a form, which is assumed to be free from the objection raised, the magnetic action being supposed to be due to a swarm of electrified corpuscles ejected by the sun. The question of energy has not, so far as I know, been discussed in this case, and it seems to be taken for granted that the total energy of the magnetic field due to a swarm of corpuscles is equal to the sum of the energies of each, calculated as if the others were not present. If the corpuscles are sufficiently far apart, this is approximately correct; but in that case the magnetic field itself would have to be negligible, except within molecular distance from each particle. How far we may go wrong by treating the energy as if it could be obtained by a process of addition becomes apparent when we consider that such treatment would lead to coefficients of self-induction which are proportional to the length of a circuit and independent of its shape.

2. If A_1, A_2, A_3 represent the components of electro-kinetic potential and u_1, u_2, u_3 the current components at any point, the total electro-magnetic energy is given by

$$T = \tfrac{1}{2}\iiint (A_1u_1 + A_2u_2 + A_3u_3)\, dx\, dy\, dz.$$

In a previous paper† I have shown that in evaluating the integral the displacement currents need not be taken into account so far as they affect the components of current, while they become important, however, in the calculation of A. If we take the axis of z to be the direction of motion, the expression for the energy reduces to

$$T = \tfrac{1}{2}\iiint Au\, dx\, dy\, dz,$$

where u stands for the convection current per unit surface within the swarm of electrons and A is the component in the direction of motion of the electro-kinetic momentum due to the whole current.

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* 'Roy. Soc. Proc.,' 1892, vol. 52, p. 307.

† 'Phil. Mag.,' 1897, vol. 43, p. 1.

The correct value for A at a point x, y, z, is*

$$\iiint u\, d\xi\, d\eta\, d\zeta \left(\frac{1}{r} - \frac{1}{2}\frac{d}{dr}\frac{z-\zeta}{r}\right).$$

where u is now supposed to be a function of ξ, η, ζ. This is to be integrated over the space occupied by the swarm, which we suppose to be cylindrical. In the calculation of A we take the two terms separately. The first is identical with the gravitational potential of matter spread with density u through the space filled by the swarm of moving electrons. As the swarm has a length which is great compared with its cross-section, the calculation may be simplified by assuming the value of A to be constant in a plane at right angles to the direction of motion. In a section at a distance h from the front, the gravitational potential due to that part of the cylinder which is in front of the section is then found to be

$$\pi u \left\{ a^2 \log\left(\frac{h}{a} + \frac{\sqrt{(h^2+a^2)}}{a}\right) + h\sqrt{(h^2+a^2)} - h^2 \right\}.$$

The second part of the integral to be determined is

$$-\tfrac{1}{2}\iiint \frac{d}{dz}\left(\frac{z-\zeta}{r}\right) d\xi\, d\eta\, d\zeta = \tfrac{1}{2}u \iiint \frac{d}{d\zeta}\left(\frac{z-\eta}{r}\right) d\xi\, d\eta\, d\zeta = -\frac{h}{2}u \iint d\eta\, d\zeta.$$

If the front portion of the cylinder alone be considered, this is equal to the gravitational potential of matter spread uniformly with surface density $\tfrac{1}{2}hu$ over the front end of the cylinder, and is therefore equal to

$$-\pi h\left(\sqrt{(h^2+a^2)} - h\right) u.$$

The two terms together therefore give for the value of A, if α be written for h/a,

$$\pi \left\{ a^2 u \log\left[\alpha + \sqrt{(1+\alpha^2)}\right] \right\}.$$

To get the electro-kinetic energy, this has to be multiplied by $\tfrac{1}{2}u$ and integrated over the cylinder. The parts of A due to the two portions of the cylinder contribute equally, and we have therefore

$$\begin{aligned} \mathrm{T} &= \pi^2 a^5 u^2 \int_0^\alpha \log\left[\alpha + \sqrt{(1+\alpha^2)}\right] d\alpha \\ &= \pi^2 a^5 u^2 \left\{ \alpha \log\left[\alpha + \sqrt{(1+\alpha^2)}\right] + 1 - \sqrt{(1+\alpha^2)} \right\}, \end{aligned}$$

where α now represents the ratio of the total length of the cylinder to the radius of its cross-section. If α be large, we may write, approximately,

$$\mathrm{T} = \pi \mathrm{V} a^2 u^2 (\log 2\alpha - 1),$$

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* 'Phil. Mag.,' 1891, vol. 32, p. 9.

where V stands for the volume of the cylinder. It is useful to introduce the magnetic force at the boundary of the cylinder, which is $H = 2\pi au$, so that

$$T = \frac{1}{4\pi} H^2 V (\log 2\alpha - 1),$$

and the energy per unit length becomes

$$T = \tfrac{1}{4} H^2 a^2 (\log 2\alpha - 1).$$

As the swarm increases in length, the energy increases in a greater proportion than the length.

3. Before discussing the results obtained, we write down the corresponding expressions for a swarm of electrons uniformly filling a sphere and moving with speed w in the direction of the axis of z. If e be the total quantity of electricity in the sphere, the magnetic energy outside the sphere is $e^2w^2/3a$.

The magnetic force inside the sphere is $e'w(x^2+y^2)^{\frac{1}{2}}/r^3$, where e' stands for the quantity of electricity contained in a sphere of radius r, which is equal to er^3/a^3. Introducing this and integrating through the volume of the sphere, we find for the total electro-kinetic energy of the moving sphere $2e^2w^2/5a$, the magnetic force at the surface in the equatorial plane being ew/a^2. If H denote this magnetic force, the energy is

$$T = \tfrac{2}{5} a^3 H^2 = \frac{3}{10\pi} H^2 V,$$

if V be the volume of the sphere.

Comparing the expressions for the cylinder with those for the sphere, it appears that in writing $H^2V/4\pi$ for the energy contained in volume V we shall underrate its value in both cases. As H is the magnetic force at the point where it reaches its greatest value, we may take the above expression to represent the smallest amount of energy which a swarm of electrified particles can contain if it occupies a volume V and causes a magnetic force H.

4. Sir Oliver Lodge* has calculated the numerical values of the dimensions of a swarm of electrons, producing a magnetic field as great as that of the storm of September 25, 1909. If the velocity of the particles be estimated at 10^9, he finds, taking into account the duration of the storm, that the square of the cross-section has the value 5×10^{20}, so that the total energy transmitted in unit time would be $4 \times 10^{28} H^2$, or, as the change in the magnetic force on this occasion was $0{\cdot}004$, we obtain finally $6{\cdot}4 \times 10^{23}$ ergs per second for the rate at which the sun would have to emit energy.

The number obtained by Lord Kelvin was $1{\cdot}5 \times 10^{36}$ in a storm which caused a magnetic force less than the tenth part of that considered by

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* 'Nature,' 1892, vol. 52, p. 307.

Sir Oliver Lodge. The difference is more apparent than real, because in the theory criticised by Lord Kelvin the whole surface of the sun was concerned in the emission of energy, while a swarm of electrons such as we are now considering would be projected only from a limited region. If, in the example chosen, the surface from which the swarm is ejected covered a circular patch subtending an angle of about 4′ of arc, as seen from the earth, we should have found for the work per unit surface of the sun a number identical with that given by Lord Kelvin.

5. The question arises whether, and in what way, the sun can supply the large electro-kinetic energy of the magnetic field which surrounds the swarm of electrified particles. If w and μ denote respectively the velocity and mass of each particle as it leaves the sun, the original energy of a number NV contained in volume V, is $\frac{1}{2}\mu \mathrm{NV} w^2$, and this energy has to provide not only the magnetic field, but also the energy which the particles retain in consequence of their own mass, or rather of the electro-kinetic energy of the magnetic field which they carry with it in their immediate surroundings.

This latter part of the energy would be $\frac{1}{2}\mu \mathrm{NV} u^2$, if u be the velocity with which they sweep past the earth. It appears therefore that u must be smaller than w, which means that the particles must lose energy between the sun and the earth. This implies a retarding force, which hitherto has been left out of account. The stream of particles moves, in fact, in a magnetic field, which is constantly increasing and thereby gives rise to an electromotive force, opposing the motion. As the stream increases in length, the electro-kinetic energy, which is due to the superposition of the magnetic fields of all the particles, soon considerably exceeds the sum of the energies of each particle taken separately, and therefore in the final stage u^2 will be small compared with w^2. In any case, we must conclude that the original energy supplied by the sun per unit volume exceeds $\mathrm{H}^2/4\pi$.

Hence

$$2\pi\mu \mathrm{N} w^2 > \mathrm{H}^2;$$

or, substituting for H the magnetic force at the cylindrical boundary,

$$w^2/u > \mathrm{H}ae/\mu.$$

Substituting for H and a the numerical values given by Sir Oliver Lodge, we find

$$w^2/u > 10^{15}.$$

It follows that, even though we may estimate the original velocity as nearly equal to that of light (provided μ is not materially increased), the velocity of the particles as they sweep past the earth is reduced to about nine kilometres a second. The passage between the sun and earth would take about a year, so that the main purpose for which the hypothesis was framed, which was to find a connecting link between disturbances on the sun and nearly simultaneous effects on the earth, would cease to be served. The difficulty might to some extent be overcome if it were possible to reduce the cross-section of the swarm, but this was fixed to suit the duration of the storm, and could not materially be diminished. It will also appear that the electrostatic repulsion between the particles puts a small cross-section out of the range of possibility.

We may pause for a moment to consider whether more favourable results might not be obtained by assuming the projected particles to have masses comparable with those of ordinary atoms of matter. It seems, however, impossible that the high velocity required could be imparted to such heavy carriers, except by a radioactive process, and examination shows that inadmissibly large quantities of radium would have to be involved, if appreciable magnetic effects are to be produced.

6. We proceed to examine the electrostatic effects, which must show themselves in the gradual expansion of the beam and in the redistribution of the electricity on the surface of the earth. The magnetic force H at a point P due to a charge e moving with velocity u is $eu \sin\theta/r^2$, if θ be the angle which the line between P and the charge forms with the direction of motion. On the other hand the electric force at P resolved in a plane at right angles to the direction of motion is $ec^2 \sin\theta/r^2$, where c is the velocity of light. Hence if E be the component of electric force at right angles to the direction of motion,

$$\mathrm{E} = \frac{c^2}{u}\mathrm{H}.$$

This relation must hold for the total effect of all current elements, provided they are all parallel. The acceleration at P of the charge e having mass μ is

$$f = \frac{c^2 e}{u\mu}\mathrm{H}.$$

We found from energy consideration that c^2/u must be greater than 10^{15}, but leaving this out of account altogether, and putting for u its greatest possible value, which is c, we may assert that

$$f > 5{\cdot}6 \times 10^{17}\mathrm{H}.$$

With 0·004 for the value of H this would give a minimum for f which is greater than 22×10^9, an acceleration sufficient to drive a particle in the first second through a distance equal to more than 8 diameters of the earth. If it be remembered that this is the acceleration at the edge of the swarm, when it has reached the earth, and that the average acceleration between the sun and the earth must have been greater, we must conclude that a swarm of electrons packed with sufficient density to cause a magnetic effect would soon get dissipated laterally into space until its magnetic action becomes negligible. The electro-magnetic attraction of the particles has not been lost sight of, but it only partly neutralises the electrostatic effect when the velocity is that of light, and with all admissible velocities it is negligible. Returning to the electric force $E = c^2H/u$, consider a point on the earth's surface where its direction is vertical. The force causes an electric field which is more than doubled by the charge induced on the earth. Again substituting the lowest possible value for $c^2/u \, (u = c)$ we find that the electric field would be at least 600H volts per centimetre or with previous value of H, 2·4 volts per centimetre. This exceeds several times the normal field of the earth, which, if the particles are negatively charged, would, on the approach of a storm, become reversed in sign and nearly doubled in intensity. No such effect is observed, but, on the contrary, one characteristic feature of atmospheric electricity is, that it remains undisturbed even by the most violent magnetic storm. If the electric force continued to act it might be argued that owing to the electric conductivity of the atmosphere the screening which would take place might explain the absence of electrostatic effects. But the small velocity of diffusion of the ions which cause the atmospheric conduction would prevent any effective screening of such sudden changes of magnetic force as are observed during magnetic storms. If the energy considerations are taken into account, the value for the electrostatic field for the same magnetic force would have to be multiplied by 100,000.*

7. The results of the previous investigation conclusively prove that magnetic storms cannot be due to a direct magnetic action of swarms of electrified particles. We must reject that theory and can do so without a pang of regret, so far as that simple variety of it is concerned, which simply considers the effect of kathode rays projected from the sun outwards, because, apart from the consideration now brought forward, the magnetic effects of such rays could not reproduce, even roughly, the characteristic features of a magnetic disturbance. It is otherwise with the more refined form in which the theory has been presented by Prof. Birkeland, who, qualitatively at any rate, has shown that an agreement might be reached, if we can imagine the particles to be drawn in towards the earth, by its magnetic forces, so that for the time being their motion is regulated by the position of the earth's magnetic poles. Nevertheless, the argument from

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* The effect of a swarm of electrified particles on the earth's electric field has already been pointed out by Dr. George Simpson ('Nature,' 1909, vol. 82, p. 37).

energy and from electrostatic considerations alike, has now been shown to be fatal to the theory in any form.

If, therefore, we wish to adhere to the hypothesis that the connexion between solar outbursts and terrestrial magnetism is due to a projection of particles by the sun, we are driven to accept the view, which I have advocated for a long time, that the particles act by increasing the ionisation of the outer regions of the atmosphere and allow the electromotive forces which are always present locally to increase the intensity of the electric circulation. The rotation of the earth, which is the primary cause of the electromotive forces which come into play, then becomes responsible for the energy.

The view that the impact of the projected particles causes luminosity remains allowable, so that Prof. Birkeland's theory of the aurora borealis is still tenable.

On the Periodicity of Sun-spots.

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(Received January 19,—Read January 26, 1911.)

In the year 1906 I presented to the Royal Society an investigation* in which it was shown that the frequency of sun-spots was subject to recurrent variations, not only in the well-known 11 years' cycle, but also in other periods, which were determined. As we now possess additional material extending over 10 years, it is interesting to examine how far the minor maxima of the subsidiary periods can be traced in the more recent records. The accompanying figure gives diagrammatically the sun-spot areas measured at Greenwich between 1898 and 1909. The numbers plotted represent the sum of the mean areas during four successive rotations, beginning with the four rotations 593/596 of Carrington's series.

There is a period of 4·79 years, which in the previous communication was shown to be persistent during the whole time covered by sun-spot records, more persistent, in fact, than that of 11 years. I have marked on the diagram the predicted times of maxima of the period with an arrow pointing upwards. The first maximum, towards the end of the year 1898, which was timed to arrive during the rotation 604, actually took place three rotations, or about 2½ months, earlier. The second maximum (September, 1903) was

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* 'Phil. Trans.,' 1906, vol. 206, p. 69.